Underwater Magnifier

Consider a symmetric bi-convex thin lens having radius R and index of refraction n_L immersed in a medium having index of refraction n. From the Lens Makers' Formula, its focal length is

$$f = \frac{R}{2}\frac{n}{n_L - n} \tag{1}$$

In air and water the lens' focal lengths are, respectively, f_A and f_W , where

$$f_A = \frac{R}{2}\frac{n_A}{n_L - n_A} \qquad\qquad f_W = \frac{R}{2}\frac{n_W}{n_L - n_W} \tag{2}$$

Their ratio is

$$\frac{f_W}{f_A} = \frac{n_W}{n_A}\left(\frac{n_L - n_A}{n_L - n_W}\right) \tag{3}$$

The **magnifying power** of this simple lens is defined as the ratio of two angles:

(1) The half-angle ϕ subtended at the eye by the lens' virtual image when the object is situated at the lens' primary focal point (the virtual image will be at infinity).
(2) The half-angle ϕ_E subtended at the eye (without the aid of the magnifying lens) when the object is at the eye's minimum focussing distance "d".

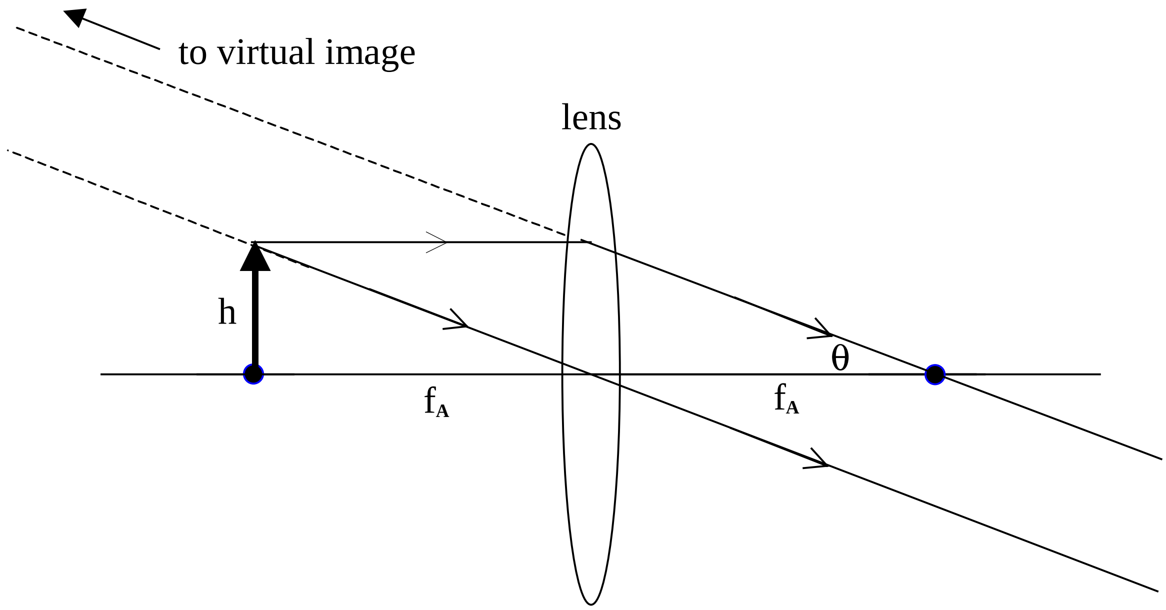
The magnifying power, M, is

$$M = \frac{\phi}{\phi_E} \tag{4}$$

We wish to compare the magnifying powers of a lens in air and underwater. In the latter case, the only complicating factor is the presence of the scuba diver's mask.

For a small object of height "h" at the distance f_A from the lens **in air** (see diagram), we have (angles in radians)

$$\phi \approx \tan\phi = \frac{h}{f_A} \tag{5}$$



Similarly, for the same object at the distance "d" in front of the unaided eye, we have

$$\phi_E \approx \tan\phi_E = \frac{h}{d} \qquad (6)$$

Thus, the magnifying power in air is

$$M_A = \frac{h/f_A}{h/d} = \frac{d}{f_A} \qquad (7)$$

For the same object and lens **in water,** the angle ϕ arises from **two** effects. In addition to the refraction by the lens, there is additional refraction by the water-air interface at the mask faceplate. The latter effect **increases** the angle. Together they produce

$$\phi = \frac{h}{f_W}\left(\frac{n_W}{n_A}\right) \qquad (8)$$

Without the magnifying lens, the largest angle at the eye is still ϕ_E=h/d (as in air) because the distance d now refers to the closest "apparent" position of the object. Hence,

$$M_W = \frac{h/f_W}{h/d}\left(\frac{n_W}{n_A}\right) = \frac{d}{f_W}\left(\frac{n_W}{n_A}\right) \qquad (9)$$

Thus,

$$\frac{M_W}{M_A} = \frac{f_A}{f_W}\left(\frac{n_W}{n_A}\right) \tag{10}$$

Substituting from (3) into (10), we finally get

$$\frac{M_W}{M_A} = \frac{n_L - n_W}{n_L - n_A} \tag{11}$$

For a glass magnifying lens with n_L=1.5, and with n_A=1.0 and n_W=1.33, the ratio becomes

$$\frac{M_W}{M_A} = 0.34$$

which indicates that the lens loses 66% of its magnifying power when moved from air into water.