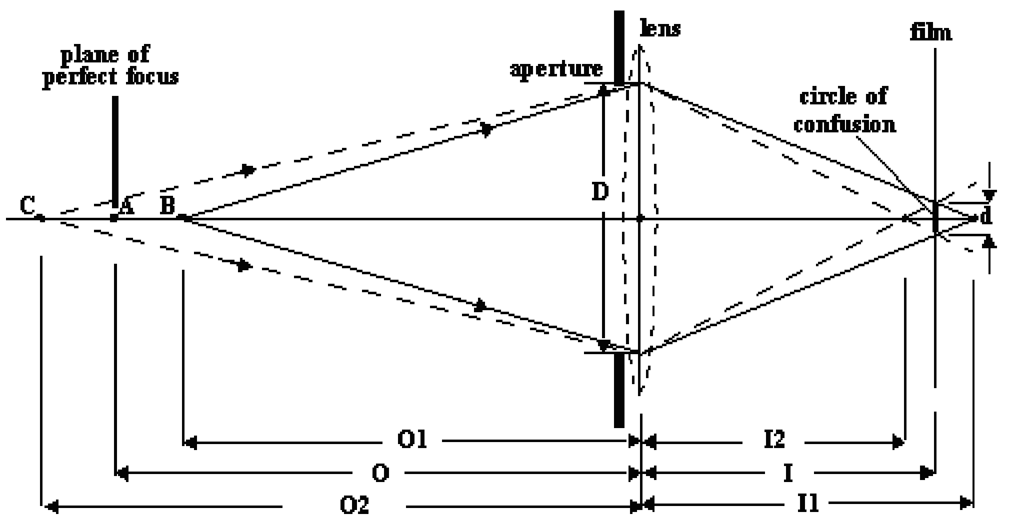
Field of Focus

When you capture a camera image on film the amount of your subject that will appear in focus depends on several factors: lens focal length, lens aperture, subject-to-camera distance, the eye's resolving power. To illustrate the relationships between these variables and the field of focus it is instructive to consider a simple camera with a lens consisting of a single element that is thin, circular, and symmetric bi-convex. We assume also that it focuses perfectly, namely, all rays from a point object converge at a point image. Although this model is much too simple to be quantitatively accurate for a typical multi-element SLR lens, it is adequate for our purposes.

In the following figure the image of the point object A is assumed to be in perfect focus at the film plane. The point objects B and C are, respectively, in front of and behind A. Because B is closer to the lens its image will be focused behind the film. Hence, at the film plane the rays from B form a blurred circle, the **circle of confusion,** of diameter d. The object point C, on the other hand,



is imaged in front of the film and (as shown) has been chosen so that it also forms a circle of confusion of diameter d at the film. If the circle of confusion is small enough so that it is perceived as a point rather than a circle, then points B and C are, respectively, on the near and far bounding planes of the field of focus.

Except for very coarse-grained films, the diameter of the maximum tolerable circle of confusion depends on how well your eyes can distinguish between two closely spaced objects. Typically, the human eye can resolve two parallel black lines on a bright clear background if they subtend an angle of 1.5 minutes of arc. Under less than ideal conditions, it is usually assumed that the eye's resolution limit is about 3.4 minutes of arc (0.001 radians). Let's assume that the image will be viewed from its **center of perspective**, meaning that the angle subtended by the image at the eye is the same as the angle subtended by the object at the camera. Blurring will therefore not be apparent until the ratio d/I (see diagram) exceeds 1/R, where R=1000, i.e. until the diameter of the circle of confusion exceeds

$$d = \frac{I}{R} \tag{1}$$

As shown below, this depends on the object distance O and the lens focal length F. Except for extreme close-ups, however, the approximation d=F/R is quite good for all object distances.

Relating object distances, images distances, and the lens focal length F by means of the usual thins lens formula, we have for the objects A, B, and C:

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F} \quad \Rightarrow \quad I = \frac{OF}{O - F} \tag{2}$$

$$\frac{1}{O1} + \frac{1}{I1} = \frac{1}{F} \quad \Rightarrow \quad O1 = \frac{(I1)F}{I1 - F} \tag{3}$$

$$\frac{1}{O2} + \frac{1}{I2} = \frac{1}{F} \quad \Rightarrow \quad O2 = \frac{(I2)F}{I2 - F} \tag{4}$$

Consider the rays from B. By similar triangles, we have

$$\frac{d}{I1 - I} = \frac{D}{I1} \quad \Rightarrow \quad I1 = \frac{DI}{D - d} \tag{5}$$

Substituting the expression for I from Equation 2 into Equation 5, and then substituting the resulting expression for I1 into Equation 3, we get

$$O1 = \frac{O}{1 + X} \tag{6}$$

where

$$X = \frac{dO}{DI} \tag{7}$$

Similarly, for the object point C we have

$$\frac{d}{I - I2} = \frac{D}{I2} \quad \Rightarrow \quad I2 = \frac{DI}{D + d} \tag{8}$$

Combining Equation 8 and Equation 2, and substituting into Equation 4, we get

$$O2 = \frac{O}{1 - X} \tag{9}$$

The field of focus extends from O1 to O2. The above results tell us a few interesting things. The extents of the field of focus in front of and behind the plane of perfect focus are given respectively by

$$O - O1 = \frac{OX}{1 + X} \quad \text{and} \quad O2 - O = \frac{OX}{1 - X} \tag{10}$$

Since X is always positive, the field of focus behind A is always wider than the field of focus in front of A. The entire width of the field of focus (**depth of field** (d.o.f.)) is therefore

$$\text{d.o.f.} = O2 - O1 = \frac{2OX}{1 - X^2} \tag{11}$$

Defining the lens **f-stop**, f=F/D, and noting from Equation (1) that the magnification ratio is M= I/O =F/(O-F), we find, to first order in the (small) ratio d/F

$$\text{d.o.f.} = 2fd\frac{M + 1}{M^2} \tag{12}$$

This tells us that for a fixed magnification ratio the depth of field is directly proportional to the magnitude of the f-stop: f/16 must give you greater depth of field than f/5.6. Also, for a fixed f-stop, increasing the magnification ratio (doing close-ups) decreases the depth of field.

It is interesting to note that for both f and M fixed, the depth of field is independent of the lens focal length. However, to first order in d/F, increasing F will increase O-O1 while decreasing O2-O. In other words, the depth of field may not change but the entire field of focus is shifted towards the lens relative to the plane of perfect focus.

For non-close-up photography it is not difficult to show that, to an excellent approximation, I-I2 and I1-I are the same and independent of the object distance. This explains why the depth-of-field marks on the lens barrel are symmetric about the central focus indicator, and why one set of marks applies to all focus distances (barrel rotations).

Is there an object distance for which **everything** behind the object is in focus? Yes, O2 in Equation 11 will be infinite when X=1. Using the definition of X (Equation 7) and Equation 2, we find that X=1 when the object distance, O, equals

$$H = \frac{F^2}{fd}(1 + \frac{d}{D}) \tag{13}$$

This is called the **hyperfocal distance**. It is also clear from Equation 10 that when X=1 the width of the near field of focus (O-O1) is H/2. So, when the object is at the hyperfocal distance and the lens is focused on it, everything from H/2 to infinity will be in focus.

REFERENCES:

Lawrence E. Mertens, "In-Water Photography", (Wiley-Interscience, 1970), chapter 6

Rudolf Kingslake, "Lenses in Photography", (Garden City Books, 1951), chapter V